SPACECRAFT DOPPLER TRACKING AS A NARROW-BAND DETECTOR OF GRAVITATIONAL WAVES

MASSIMO TINTO
Jet Propulsion Laboratory, MS 161-260, Pasadena,
CA 91109, USA

We discuss spacecraft Doppler tracking for detecting gravitational waves in which Doppler data recorded on the ground are linearly combined with Doppler measurements made on board a spacecraft. By using the four-link radio system first proposed by Vessot and Levine [1], we derive a new method for removing from the combined data the frequency fluctuations due to the Earth troposphere, ionosphere, and mechanical vibrations of the antenna on the ground. This method also reduces the frequency fluctuations of the clock on board the spacecraft by several orders of magnitude at selected Fourier components, making Doppler tracking the equivalent of a *xylophone* detector of gravitational radiation[2]. In the assumption of calibrating the frequency fluctuations induced by the interplanetary plasma, a strain sensitivity equal to 4.7 $\times 10^{-18}$ at 10^{-3} Hz is estimated.

1 Introduction

Doppler tracking of interplanetary spacecraft is the only existing technique that allows searches for gravitational radiation in the millihertz frequency region[3]. The frequency fluctuations induced by the intervening media have severely limited the sensitivities of these experiments. Among all the propagation noise sources, the troposphere is the largest and the hardest to calibrate to a reasonably low level. Its frequency fluctuations have been estimated to be as large as 10^{-13} at 1000 seconds integration time[4].

In order to systematically remove the frequency fluctuations due to the troposphere in the Doppler data, it was pointed out by Vessot and Levine[1] and Smarr *et* al. [5] that by adding to the spacecraft payload a highly stable frequency standard, a Doppler read-out system, and by utilizing a transponder at the ground antenna, one could make Doppler one-way (Earth-to-spacecraft, spacecraft-to- Earth) as well as two-way (spacecraft- Earth-spacecraft, Earth-spacecraft- Earth) measurements. This way of operation makes the Doppler link totally symmetric and allows the complete removal of the frequency fluctuations due to the Earth troposphere, ionosphere, and mechanical vibrations of the ground antenna by properly combining the Doppler data recorded on the ground with the data measured on the spacecraft. Their proposed scheme relied on the possibility of flying a hydrogen maser on a dedicated mission. Although current designs of Hydrogen masers have demanding requirements in mass and power consumption, it seems very likely that by the beginning of the next century new space-qualified atomic clocks, with frequency stability of a few parts in 10^{-16} at 1000 seconds integration time, will be available. They would provide a sensitivity gain of almost a factor of one thousand with respect to the best performance crystal-driven oscillators. Although this clearly would imply a great improvement in the technology of space born clocks, it would not allow us to reach a Doppler sensitivity better than a few parts 10^{-16}. This would be only a factor of five or ten better than the Doppler sensitivity expected to be achieved on the future Cassini project, a NASA mission to Saturn, which will take advantage of a high radio frequency link (32 GHz) in order to minimize the plasma noise, and will use a purposely built water vapor radiometer for calibrating up to ninety five percent the frequency fluctuations due to the troposphere.

in this paper we adopt the radio link configuration first envisioned by Vessot and Levine?, but we combine the Doppler responses measured on board the spacecraft and on the ground in a different way, as it will be shown in the following sections. Furthermore our technique allows us to reduce by several orders of magnitude, at selected Fourier components, the noise due to the clock on board the spacecraft[2].

2 Doppler Tacking as a Narrow-band Detector

in Doppler **tracking experiments a distant** interplanetary spacecraft is monitored from Earth through a radio link, and the Earth and the spacecraft act as free-falling test particles. In one-way Doppler measurements a radio signal of nominal frequency $\tilde{\nu}_0$ referenced to an onboard clock is transmitted to Earth, where it is compared to a signal referenced to a highly stable clock. In *two*-way operations, which have been used so far in gravitational wave searches, a radio signal of frequency ν_0 is transmitted to the spacecraft, and coherently transponder back to Earth.

If a Doppler readout system is added to the spacecraft radio instrumentation, and a transponder is installed at the ground station, one-way as well as two-way Doppler data can also be recorded on board the spacecraft. If we assume the Earth clock and the onboard clock to be synchronized, then the one-way and two-way Doppler data measured at time t on the Earth ($E_1(t)$, $E_2(t)$ respectively), and the one-way and two-way Doppler measured at the same time t on the spacecraft ($S_1(t), S_2(t)$), are given by tile following rather complete expressions[2]

$$E_1(t) = \frac{(1-\mu)}{2}[h(t-(1+\mu)L) - h(t)] + C_{sc}(t-L) - C_E(t)$$
$$+ T(t) + B(t-L) + A_{sc}(t-L) + EL_{E_1}(t) + P_{E_1}(t)\ , \quad (1)$$

$$
\begin{aligned}
E_2(t) = & -\frac{(1-\mu)}{2} h(t) - \mu h(t-(1+\mu)L) + \frac{(1+\mu)}{2} h(t-2L) \\
& + C_E(t-2L) - C_E(t) + 2B(t-L) + T(t-2L) + T(t) \\
& + A_E(t-2L) + A_{sc}(t-L) + TR_{sc}(t-L) + EL_{E_2}(t) \\
& + P_{E_2}(t)
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
S_1(t) = & \frac{(1+\mu)}{2}[h(t-L) - h(t-\mu L)] + C_E(t-L) - C_{sc}(t) \\
& + T(t-L) + B(t) + A_E(t-L) + EL_{S_1}(t) + P_{S_1}(t) \ ,
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
S_2(t) = & -\frac{(1+\mu)}{2} h(t-\mu L) + \mu h(t-L) + \frac{(1-\mu)}{2} h(t-2L-\mu L) \\
& + C_{sc}(t-2L) - C_{sc}(t) + 2T(t-L) + B(t-2L) + B(t) \\
& + A_{sc}(t-2L) + A_E(t-L) + TR_E(t-L) + EL_{S_2}(t) \\
& + P_{S_2}(t) \ ,
\end{aligned}
\tag{4}
$$

where $h(t)$ is equal to

$$
h(t) = h_+(t)\cos(2\phi) + h_\times(t)\sin(2\phi) \ , \tag{5}
$$

and is the gravitational wave signal. In Eqs. (1 - 4) μ is the cosine of the angle between the direction of propagation of the wave and the line of sight to the spacecraft, $h_+(t)$, $h_\times(t)$ are the wave's two independent amplitudes referenced to a given set of axes defined in the plane of the wave, ϕ is the polar angle describing the projection of the direction to the spacecraft in the plane of the wave, and L is the distance to the spacecraft (units in which the speed of light $c = 1$)

In Eqs. (1 -4) $C_E(t)$ and $C_{sc}(t)$ represent the fluctuations due to the ground and onboard clock respectively, $B(t)$ is the effect of the spacecraft buffeting, $TR_{sc}(t)$ and $TR_E(t)$ represent the noise due to the transponder on board and on the ground respectively, $EL_{E_1}(t)$, $EL_{E_2}(t)$, $EL_{S_1}(t)$, and $EL_{S_2}(t)$ the noises from the electronics at the ground station and on the spacecraft in the one-way and two-way data, and $P_{E_1}(t)$, $P_{E_2}(t)$, $P_{S_1}(t)$, and $P_{S_2}(t)$ the frequency fluctuations due to the interplanetary plasma. The plasma noise can be entirely calibrated by using dual frequencies[6]. The Doppler data $S_1(t)$ and $S_2(t)$ are then time tagged, and telemetered back to Earth in real time or at a later time during the mission.

It was first pointed out by Vessot and Levine[7] that by properly combining some of the four Doppler data streams it was possible to calibrate the frequency fluctuations of the troposphere, ionosphere, and ground antenna noise, $T(t)$. We have further improved their result[2] by showing that there exists a unique linear combination of the two one-way Doppler data which does not contain $T(t)$ and minimizes the r.m.s. noise level. The corresponding expression $\widetilde{y}(f)$ of this linear combination in the Fourier domain is equal to [2]

$$\widetilde{y}(f) = \frac{1}{2} \left[\widetilde{S_1}(f) - \widetilde{E_1}(f)\, e^{2\pi i f L}\right] . \tag{6}$$

If we substitute the Fourier transforms of Eqs. (1, 3) into Eq. (6) we get

$$\begin{aligned}
\widetilde{y}(f) &= \frac{e^{2\pi i f L}}{2} \left[1 - \frac{(1+\mu)}{2}\, e^{2\pi i f(\mu-1)L} - \frac{(1-\mu)}{2}\, e^{2\pi i f(\mu+1)L}\right] \widetilde{h}(f) \\
&+ \widetilde{C_E}(f)\, e^{2\pi i f L} - \frac{1}{2}\widetilde{C_{sc}}(f) \left[e^{4\pi i f L} + 1\right] + \frac{1}{2}\widetilde{B}(f) \left[1 - e^{4\pi i f L}\right] \\
&+ \frac{e^{2\pi i f L}}{2} \left[\widetilde{A_E}(f) - \widetilde{A_{sc}}(f)\, e^{2\pi i f L}\right] + \frac{1}{2} \left[\widetilde{P_{S_1}}(f) - \widetilde{P_{E_1}}(f)\, e^{2\pi i f L}\right] \\
&+ \frac{1}{2} \left[\widetilde{EL_{S_1}}(f) - \widetilde{EL_{E_1}}(f)\, e^{2\pi i f L}\right] .
\end{aligned} \tag{7}$$

Eq. (7) shows that the transfer function of the noise of the onboard clock, $\widetilde{C_{sc}}(f)$ can be minimized at multiple integers of the inverse of twice the round trip light time. The sensitivity figures achievable with this technique have been estimated in reference [2] under several radio hardware configurations. The lowest sensitivity that this technique can achieve at 10^{-3} Hz has been estimated to be equal to 4.7×10^{-18}, under the assumption that the inverse of the round trip light time does not change more than the frequency resolution Δf of the data over forty days. In order to show that this is not an unrealistic hypothesis, we have analyzed the Cassini trajectory when gravitational wave experiments will be performed. We have found that the distance to the spacecraft can be approximated quite accurately by the following quadratic function of time

$$\begin{aligned}
L(t) &= \alpha + \beta\, t + \gamma\, t^2 , \\
\alpha &= 2.9 \times 10^{3} \text{ sec.} ; \ \beta = -1.4 \times 10^{-5} ; \ \gamma = 1.1 \times 10^{-**} \text{ Hz} ,
\end{aligned} \tag{8}$$

where the numerical values for $\alpha, \beta,$ and γ given in Eq. (8) correspond to the first Cassini solar opposition. The time derivative of Eq. (8), calculated for a time interval of forty days, implies the following maximum variation δf of the frequency $f_1 = 1/4L$

$$\frac{\delta f}{f_1} = 6.3 \times 10^{-5} < \frac{\Delta f}{f_1} = 1.6 \times 10^{-3} . \tag{9}$$

3 conclusions

The main result of our analysis shows that by flying a frequency reference and by adding a Doppler extractor on board a spacecraft and a transponder at the ground antenna, we can achieve at selected Fourier components a strain sensitivity of 4.7 x 10^{-18}. Our method relies on a properly chosen linear combination of the one-way Doppler data recorded on board with those measured on the ground. It allows us to remove entirely the frequency fluctuations due to the troposphere, ionosphere, and antenna mechanical, and to reduce by several orders of magnitude the noise due to the onboard clock.

The experimental technique presented in this paper can be extended to a configuration with two spacecraft tracking each other through microwave or laser links. Future space-based laser interferometric detectors of gravitational waves?, for instance, could implement this technique as a backup option, if failure of some of their components would make the normal interferometric operation impossible.

4 Acknowledgements

This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

References

1. R.F.C. Vessot and M.W. Levine, *Gen. Relativ.* Gr-suit. **10,** 181 (1979).
2. M. Tinto, *Phys. Rev. D,* **10,** 5354, (1996).
3. F. B. Estabrook and H.D. Wahlquist, Gen. *Rel. Grav.*, **6** 439, 1975.
4. J.W. Armstrong. In: ***Gravitational Wove*** Data Analysis, ed. Schutz, B. F., p.153 (1989), (Dordrecht, Kluwer).
5. L.L. Smarr, R.F.C. Vessot, C.A. Lundquist, R. Decher, and T. Piran, *Gen. Relativ. Gravit.* **15,** 2 (1983).
6. A. L. Riley, D. Antsos, J.W. Armstrong, P. Kinman, H. D. Wahlquist, B. Bertotti, G. Comoretto, B. Pernice, G. Carnicella, and R. Giordani, *Jet Propulsion **Laboratory Report,*** Pasadena, California, January 22, (1990).
7. LISA: Laser Interferometer Space Antenna for the detection and observation of gravitational waves. A Cornerstone Project in ESA's long term space science program *Horizon 2000 Plus,* **Pre-Phase** A **Report, MPQ** 208, (December 1995).